

On the Vaz no horizon black hole

Hristu Culetu,
Ovidius University, Dept.of Physics and Electronics,
B-dul Mamaia 124, 900527 Constanta, Romania,
e-mail : hculetu@yahoo.com

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Abstract

We propose a no horizon black hole whose collapsing matter condenses close to the event horizon and before its formation. Compared to Vaz's model [4], our interior geometry depends only on one parameter r_0 - the radius of the region where quantum fluctuations are significant. While the equation of state of the inner fluid is $p_r = -\rho$ and the traverse pressures are vanishing, the surface stress tensor corresponds to an anisotropic fluid with negative surface tension. Using the junction conditions on the boundary of the collapsing star, we found that r_0 is half its Schwarzschild radius and not $2M$, as previously obtained by Vaz for a dust ball.

1 Introduction

Although the classical collapse process suggest that an enough massive star will undergo collapse until a singularity forms, the picture deeply changes when quantum gravity (QG) is taken into account [1, 2, 4, 5]. It is supposed QG effects will play a crucial role in determining the outcome of gravitational collapse during its final stages. The semiclassical analysis would suggest that information is lost if the black hole (BH) evaporates completely through Hawking radiation. Therefore, the thermal evaporation mechanism leaves probably behind a stable remnant that contains all the information falling into the BH. Hawking [6] has recently expressed objections to the AMPS firewall [7, 8] and suggested that the correct solution of the AMPS paradox is that event horizons do not form but only apparent horizons. Moreover, Mersini-Houghton [3] shows that due to the negative energy Hawking radiation, the collapse of the star stops at a finite radius before the singularity and the BH horizon have formed. The star bounces instead of collapsing to a BH.

Vaz [4] proposed a resolution to the AMPS paradox, considering that the collapsing matter does not undergo continuous collapse to a singularity but condenses on the apparent horizon of the BH. He constructed static solutions with no tangential pressures. Every infalling shell of dust is accompanied by

the emission of a positive energy shell from the center of the collapsing stellar object and this process of energy extraction from the center continues until the collapse terminates, describing the effect of strong quantum fluctuations close to the center [4].

Our starting point in this paper is Vaz's quasi-classical configuration, namely a spherically-symmetric source that occupies a finite region. Instead of using a dust cloud that condenses into the apparent horizon, our fluid stress tensor possesses a radial pressure $p_r = -\rho$ where ρ is its energy density, with no tangential pressures. The metric inside the collapsing "plasma ball" is simple, with a curvature singularity at $r = 0$ which, however, is not a part of the spacetime (we have taken, as Vaz [4] did, $r \geq r_0$, where r_0 is the region where quantum fluctuations are expected to dominate. In addition, we introduce a surface stress tensor on the outer boundary r_b of the star, for the junction conditions to hold. Our proposal leads to the same relation $r_b = 2M + r_0$ between the boundary radius r_b and the Schwarzschild mass M of the star but, however, we found that $r_0 = M$ and not $r_0 = 2M$ as the author of [4] obtained for his dust ball.

2 Interior metric

Vaz [4] already found the solutions of Einstein's equations (without Λ) corresponding to a source located in a finite region. He did not impose any equation of state between the energy density ρ and the radial pressure p_r and set the trasversal pressures to zero. However, his interior geometry depends on an extra parameter γ and p_r has a complicate dependence on the radial variable r . To make things more simple, we propose

$$p_r = -\rho \quad (2.1)$$

as the equation of state of the fluid inside the collapsing star. It is not the case to repeat the steps used by Vaz for obtaining the expressions for ρ and p_r and the corresponding interior geometry. With the extra restriction (2.1) the following inner geometry is obtained

$$ds^2 = -\frac{r_0}{r}dt^2 + \frac{r}{r_0}dr^2 + r^2d\Omega^2 \quad (2.2)$$

where $d\Omega^2$ stands for the metric on the unit two-sphere and r_0 , as we shall see, is related to the domain where the quantum fluctuations from the central region are significant. As it was noticed in [4], the singularity at $r = 0$ does not create problems as the solution is valid for $r \geq r_0$ only.

The energy-momentum tensor looks now as

$$T^a_b = \text{diag} \left(-\frac{1}{8\pi r^2}, -\frac{1}{8\pi r^2}, 0, 0 \right) \quad (2.3)$$

where a, b run from 0 to 3, $\rho = -p_r = 1/8\pi r^2$, $p_\theta = p_\phi = 0$. Let us note that T^a_b from (2.3) satisfies all the energy conditions and ρ and g_{rr} acquire the same

expressions as those obtained by Vaz because to get their values the expression of g_{tt} is nowhere used. We also observe that, although the equation of state is of de Sitter type, the metric (2.2) is not de Sitter, due to the vanishing of the tangential pressures. In the domain of interest ($r \geq r_0$) the spacetime (2.2) has no singularities and no horizons. For example, the Ricci scalar is $R^a_a = 2/r^2$ and the Kretschmann scalar is $R_{abcd}R^{abcd} = 4(r^2 - 2r_0r + 3r_0^2)/r^6$, with a maximum value of $8/r_0^4$ at $r = r_0$. Moreover, the radial acceleration of a static observer is $a^r = -r_0/2r^2 < 0$, so that the gravitational field is repulsive.

The total mass $m(r)$ by the radius r is given by [9]

$$m(r) = \int 4\pi r^2 \rho(r) dr = \frac{r}{2} \quad (2.4)$$

as if inside any sphere of radius r were a BH with mass $m(r)$. As far as the Komar energy is concerned, we obtain [10, 11]

$$W = 2 \int (T_{ab} - \frac{1}{2}g_{ab}T^c_c)u^a u^b N \sqrt{\det\gamma} d^3x = 0, \quad (2.5)$$

due to the contribution of the negative radial pressure. $N = \sqrt{-g_{tt}}$ is the lapse function, $u^a = (\sqrt{r/r_0}, 0, 0, 0)$ is the velocity vector field of a static observer and $\det\gamma$ represents the determinant of the spatial metric.

The Misner-Sharp mass M_{ms} is obtained from [12, 13, 14]

$$1 - \frac{2M_{ms}}{r} = g^{ab}\nabla_a r \nabla_b r, \quad (2.6)$$

which gives us

$$M_{ms} = \frac{r}{2} - \frac{r_0}{2}. \quad (2.7)$$

If we denote r_b the radius of the boundary of the star, we have $M_{ms}(r_b) = r_b/2 - r_0/2$ which, as we shall see, gives exactly the Schwarzschild mass M measured by an outer observer. In other words, the total mass [9] $m(r) = M_0 + M_{ms}$, so that $M_0 = r_0/2$ gives the (negative) contribution of the central mass [4].

3 Junction conditions

As we mentioned above, r_b gives the outer boundary of the collapsing star. Our next step is to match the interior spacetime (2.2) to the exterior empty space described by the Schwarzschild geometry

$$ds^2 = -(1 - \frac{2M}{R})dT^2 + \frac{1}{1 - \frac{2M}{R}}dR^2 + R^2d\Omega^2 \quad (3.1)$$

where M is the Schwarzschild mass, measured from large distances. Equating the 1st fundamental forms of (2.2) and (3.1), one obtains

$$\frac{r_0}{r_b}dt^2 = (1 - \frac{2M}{R_b})dT^2, \quad \frac{r_b}{r_0}dr^2 = \frac{1}{1 - \frac{2M}{R_b}}dR^2, \quad r_b = R_b \quad (3.2)$$

whence

$$\sqrt{\frac{r_0}{r_b}} t_b = \sqrt{1 - \frac{2M}{R_b}} T_b, \quad \sqrt{\frac{r_b}{r_0} \left(1 - \frac{2M}{r_0}\right)} = 1. \quad (3.3)$$

Using the 2nd relation (3.3) we conclude that $T_b = t_b$ and therefore we have

$$r_b = r_0 + 2M \quad (3.4)$$

i.e. the same condition as that inferred by Vaz [4] (his Eq. 24), but using a different interior geometry. Eq. (3.4) shows that the star radius r_b is always greater than the Schwarzschild radius, even at the final stages of the collapsing process. The difference is exactly the radius of the region of strong quantum fluctuations.

For an estimation of r_0 we call on the 2nd junction condition - the relation between the jump of the extrinsic curvature when the boundary is crossed and the surface stress tensor (the Lanczos equation)

$$[K_{ab}] - h_{ab}[K^c_c] = -8\pi S_{ab} \quad (3.5)$$

where $[K_{ab}] = K_{ab}^+ - K_{ab}^-$ is the jump of the extrinsic curvature of the $r = r_b$ boundary Σ , $h_{ab} = g_{ab} - n_a n_b$ is the induced metric on Σ and n_b is the normal on Σ , with $n_b n^b = 1$. The extrinsic curvature tensor is given by

$$K_{ab}^\pm = h^c_a \nabla_c n_b^\pm, \quad (3.6)$$

where $+$ ($-$) refers to the exterior (interior) geometries. With $n_a^- = (0, \sqrt{r/r_0}, 0, 0)$, the inner geometry gives us

$$K_{tt}^- = \frac{r_0}{2r^2} \sqrt{\frac{r_0}{r}}, \quad K_{\theta\theta}^- = \sqrt{r_0 r}, \quad K_a^{-,a} = \frac{3}{2r} \sqrt{\frac{r_0}{r}}, \quad (3.7)$$

evaluated at $r = r_b$. With $n_a^+ = (0, 1/\sqrt{1 - \frac{2M}{r}}, 0, 0)$, the outer Schwarzschild geometry yields

$$K_{TT}^+ = -\frac{M}{R^2} \sqrt{1 - \frac{2M}{R}}, \quad K_{\theta\theta}^+ = R \sqrt{1 - \frac{2M}{R}}, \quad K_a^{+,a} = \frac{2M - 3R}{R^2 \sqrt{1 - \frac{2M}{R}}}, \quad (3.8)$$

evaluated at $r = r_b$. The jump of the extrinsic curvature when Σ is crossed yields

$$[K_a^a] = \frac{1}{2r_b} \sqrt{\frac{r_b}{r_0}}. \quad (3.9)$$

In addition, we have

$$[K_{tt}] = -\frac{1}{2r_b} \sqrt{\frac{r_0}{r_b}}, \quad [K_{\theta\theta}] = 0. \quad (3.10)$$

The next step is to propose a stress tensor on the boundary Σ . Let S_{ab} be given by the expression

$$S_{ab} = (p_s + \sigma) u_a u_b + p_s h_{ab} + \pi_{ab}, \quad (3.11)$$

where $p_s = -\tau$ is the surface pressure, σ is the surface energy density, τ is the surface tension and π_{ab} is the anisotropic stress tensor, with $\pi_a^a = 0$, $\pi_b^a n^b = 0$. We consider the surface fluid has to be anisotropic to match the interior of the "plasma ball", where $p_\theta = 0$. By means of the equations (3.5), (3.9) and (3.10), one obtains

$$S_{tt} = 0, \quad 8\pi S_{\theta\theta} = \frac{r_b}{2} \sqrt{\frac{r_b}{r_0}}. \quad (3.12)$$

Let us assume now that

$$p_s + \sigma = 0 \quad (3.13)$$

is the equation of state of the surface fluid. It is justified by the repulsive character of the inner gravitational field which led to $a^r < 0$, i.e. to keep a test particle at rest we must act on it with a force toward the center. Using now (3.12) and (3.13) in (3.11), we get the set of equations

$$8\pi(r_b^2 p_s + \pi_{\theta\theta}) = \frac{r_b}{2} \sqrt{\frac{r_b}{r_0}}, \quad \pi_{tt} - \frac{r_0}{r_b} p_s = 0, \quad \pi_t^t + 2\pi_\theta^\theta = 0, \quad (3.14)$$

which leads to

$$p_s = -\sigma = -\pi_t^t = 2\pi_\theta^\theta = \frac{1}{24\pi r_b} \sqrt{\frac{r_b}{r_0}} \quad (3.15)$$

Let us make use now of the Young-Laplace equation [15, 11]

$$-[p_r] = \tau K_a^a. \quad (3.16)$$

With $[p_r] = p_{r,out} - p_{r,in} = 1/8\pi r^2$ from (2.3), $\tau = \sigma$ from (3.15) and K_a^a from (3.8), we obtain

$$\sqrt{\frac{r_b}{r_0}}(2r_b - 3M) = 3r_b \sqrt{\frac{r_0}{r_b}} \quad (3.17)$$

whence

$$2r_b - 3M = 3r_0. \quad (3.18)$$

Solving for r_0 in (3.4) and (3.18), we finally come to

$$r_0 = M. \quad (3.19)$$

In other words, r_0 is half the gravitational radius of the Schwarzschild mass M . In terms of the mass M the surface energy density and surface pressure become

$$p_s = -\sigma = \frac{1}{24\pi\sqrt{3}M}. \quad (3.20)$$

For the Misner-Sharp mass we get $M_{ms} = r_b/2 - r_0/2 = M$ and the total mass $m(r_b) = r_b/2 = 3M/2$. To summarize, the negative central mass $M_0 = r_0/2$ is the difference between the total mass $m(r_b)$ till the boundary and the Misner-Sharp mass. We note that our result does not coincide with $r_0 = r_b/2$ obtained by Vaz [4]. His estimation of r_0 is based on how much energy is extracted from the center during the star collapse (every collapsing shell is

accompanied by an inner, outgoing wave which will extract energy from the center). However, in the calculation of the average energy of the outgoing shell, Vaz used a power series developing which seems, in our opinion, to be debatable. For example, the r.h.s. of Eq. (27) of [4] is obtained in the limit the shell spacing $\sigma \rightarrow 0$. More precisely, the small parameter is considered to be σr_i so that, actually, $\sigma r_i \ll 1$ (private communication from the author of [4]). But we must take in fact $\sigma r_i / l_{Pl}^2 \ll 1$, for to use a dimensionless expansion parameter (l_{Pl} is here the Planck length). But it is hard to conceive a realistic shell to obey such an inequality, with l_{Pl}^2 at the denominator. Therefore, we consider the region of strong quantum fluctuations occupies half the Schwarzschild radius of the collapsing star.

4 Conclusions

Recently Hawking [6] raised several objections to the firewall formation during the collapse of a black hole. The final stage of the collapsing process is not the central singularity but only an apparent horizon forms. Based on Hawking conjecture, Vaz [4, 5] considers dust collapse that terminates on the apparent horizon. In his model the collapse wave function indicates that there is a process by which energy extraction from the center takes place.

We proposed in this paper a similar mechanism for a no horizon black hole formation by means of a simple interior geometry where the equation of state of the fluid is $p_r + \rho = 0$. To satisfy the junction conditions on the boundary of the "plasma ball", we introduced a surface stress tensor corresponding to an anisotropic fluid. Using the Young-Laplace equation as an extra condition, we found that the radius r_0 of the region where there are strong quantum fluctuations is half the gravitational radius of the Schwarzschild BH, contrary to Vaz's result that $r_0 = 2M$.

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